Kazuhiko Kurano*, School of Science and Technology, Meiji University, Higashimita 1-1-1, Tama-ku, Kawasaki, Kanagawa 214-8571, Japan. Rationality of the negative curve and finite generation of symbolic Rees rings.
Let $K$ be a field. Let $a, b, c$ be pairwise coprime positive integers such that $\sqrt{a b c} \notin \mathbb{N}$. Let $X$ be the weighted projective space $\operatorname{Proj}(K[x, y, z])$ with $\operatorname{deg}(x)=a, \operatorname{deg}(y)=b, \operatorname{deg}(z)=c$, respectively. Let $f: Y \rightarrow X$ be the blow-up at the smooth point defined by the kernel $P$ of the $K$-algebra map $K[x, y, z] \rightarrow K[T]$ defined by $x \mapsto T^{a}, y \mapsto T^{b}, z \mapsto T^{c}$. Let $E$ be the exceptional divisor. If the symbolic Rees ring $R_{s}(P)$ (equivalently, the Cox ring of $Y$ ) is Noetherian, there exists a curve $C(\neq E)$ such that $C^{2}<0$.

In this talk, we give some sufficient condition for the negative curve to be rational. All examples (that I know) of negative curves satisfy this condition. Therefore, I do not know any examples of non-rational negative curves.

Assume that there exists a negative curve $C$. Then $R_{s}(P)$ is Noetherian if and only if there exists a curve $D$ on $Y$ such that $C \cap D=\emptyset$. (The defining equations of $C$ and $D$ satisfy the Huneke's criterion for finite generation.) In the case where $C$ is rational, it is possible to estimate the degree of $f(D)$. Using computers, it is possible to determine whether $R_{s}(P)$ is Noetherian in some cases. (Received January 19, 2019)

