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Brandon Alberts* (blalberts@math.wisc.edu). *Certain Unramified Metabelian Extensions Using Lemmermeyer Factorizations.*

We study solutions to the Brauer embedding problem with restricted ramification. More specifically, suppose G and A are finite abelian groups, E is a central extension of G by A , and $f : \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \rightarrow G$ is a continuous homomorphism. We determine conditions on the discriminant of f that are equivalent to the existence of an unramified lift $\tilde{f} : \text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \rightarrow E$ of f .

As a consequence, we use conditions on the discriminant of an abelian extension K/\mathbb{Q} to classify unramified extensions L/K normal over \mathbb{Q} where the (nontrivial) commutator subgroup of $\text{Gal}(L/\mathbb{Q})$ is contained in its center. This generalizes a result due to Lemmermeyer stating that the quadratic field of discriminant d , $\mathbb{Q}(\sqrt{d})$, has an unramified extension $M/\mathbb{Q}(\sqrt{d})$ normal over \mathbb{Q} with $\text{Gal}(M/\mathbb{Q}(\sqrt{d})) = H_8$ (the quaternion group) if and only if the discriminant factors $d = d_1 d_2 d_3$ into a product of three coprime discriminants, at most one of which is negative, satisfying

$$\left(\frac{d_i d_j}{p_k} \right) = 1$$

for each choice of $\{i, j, k\} = \{1, 2, 3\}$ and prime $p_k \mid d_k$. (Received February 12, 2018)