The aim is to give an overview of selected results from complex analysis that can be understood geometrically in terms of the hyperbolic metric. Hyperbolic geometry was created axiomatically in the first half of the 1800s. Its role in complex analysis begins with the introduction of the Poincaré model for hyperbolic geometry on the unit disk (or upper half-plane). This model is differential geometric in nature as it starts with the hyperbolic (or Poincaré) metric with curvature $-1$ on the unit disk. The focus of this talk is the hyperbolic metric, rather than hyperbolic geometry. From this perspective, the first major contribution is due to Georg Pick in 1915, the Schwarz-Pick Lemma. The next major contribution is due to Lars Ahlfors in 1938 who established the fundamental theorem that the hyperbolic metric is maximal among all conformal metrics with curvature at most $-1$. The main portion of the talk is on some developments since the 1950s. The hyperbolic metric continues to play an important role in geometric function theory and in suggesting generalizations to other contexts. (Received July 27, 2018)