Order reduction, i.e., the convergence of the solution at a lower rate than the formal order of the time-stepping scheme, is a fundamental problem in stiff ODEs, and particularly in PDE IBVPs. Runge-Kutta schemes with high stage order provide a remedy, but unfortunately high stage order (above two) is incompatible with DIRK schemes. In this talk we present the concept of weak stage order, and demonstrate (a) how it can address order reduction in important PDE problems; and (b) that it can recover up to fourth order convergence with DIRK schemes. (Received July 30, 2018)