The Sergeev duality states that the action of the Type Q Lie superalgebra $q(n)$ and the Sergeev algebra fully centralize each other on the tensor space. Hill-Kujawa-Sussan (2011) generalized this work to the one boundary setting. We further study the two boundary generalization and define the degenerate two boundary affine Hecke-Clifford algebra $B_d$ using generators and relations. It admits a $q(n)$-linear action on $M \otimes N \otimes V^\otimes d$ for the natural representation $V$ and arbitrary $q(n)$-modules $M$ and $N$. When $M$ and $N$ are polynomial modules parametrized by a staircase and a single row partition, respectively, the action of $B_d$ factors through a quotient algebra $H_d$. Using combinatorial tools such as the Bratteli diagram and shifted Young tableaux, we construct simple modules for $H_d$. These modules occur as irreducible $H_d$-summands of $M \otimes N \otimes V^\otimes d$. (Received July 16, 2018)