A structure theorem and natural extensions for $p$-adic scaling maps.

Many standard examples of transformations on the $p$-adic integers are $p^n$-Lipschitz on each ball of radius $p^{-n}$. We show that maps with this scaling property can be expressed as an iterate of the one-sided shift composed with an isometry. Then we use this decomposition to give a formula for the natural extension. (Received August 27, 2018)