Hessenberg varieties have become a powerful tool for tackling combinatorial problems, but Hessenberg varieties themselves are also intriguing combinatorial objects. A Hessenberg variety is a collection of full flags determined by an element $X$ in the Lie algebra $\mathfrak{g}$ and a Hessenberg space $H$ which is an upper order ideal in the root decomposition of $\mathfrak{g}$. By fixing $X$ but varying $H$, the Hessenberg varieties form a poset of algebraic varieties, ordered by containment. The analogous poset for Schubert varieties is isomorphic to the Bruhat order, but for Hessenberg varieties, the containment poset is a coarsening of the poset of upper order ideals. This talk will discuss recent results about the structure of these posets of Hessenberg varieties. (Received May 29, 2018)