Lauren Keough*, keoulaur@gvsu.edu, and David Shane. Toward A Nordhaus-Gaddum Inequality for the Number of Dominating Sets.

A dominating set in a graph $G$ is a set $S$ of vertices such that every vertex of $G$ is either in $S$ or is adjacent to a vertex in $S$. Nordhaus-Gaddum inequalities relate a graph $G$ to its complement $\bar{G}$. For example, the original Nordhaus-Gaddum inequalities were about the sum and product of the chromatic number of a graph and the chromatic number of its complement. In this spirit Wagner proved that any graph $G$ on $n$ vertices satisfies $\partial(G) + \partial(\bar{G}) \geq 2^n$ where $\partial(G)$ is the number of dominating sets in a graph $G$. In the same paper he comments that an upper bound for $\partial(G) + \partial(\bar{G})$ among all graphs on $n$ vertices seems to be much more difficult. We conjecture that the complete balanced bipartite graph maximizes $\partial(G) + \partial(\bar{G})$ and have verified this computationally for all graphs on at most 10 vertices. We’ll prove an upper bound on $\partial(G) + \partial(\bar{G})$, provide a maximum and minimum degree condition on the extremal graphs, and discuss some of the other techniques we tried to shed some light on why this problem seems difficult. (Received August 15, 2018)