1143-05-420 **Zoltan Furedi*** (z-furedi@math.uiuc.edu) and **Imre Ruzsa**. *Nearly-subadditive sequences.*

The sequence $a(1), a(2), a(3), \ldots$ of reals is called *subadditive* if $a(n+m) \leq a(n) + a(m)$ holds for all integers $n, m \geq 1$. Fekete's lemma (1923) states that the sequence $\{a(n)/n\}$ has a limit (possible negative infinity). Let $f(n)$ be a non-negative, non-decreasing sequence. deBruijn and Erdős (1952) called the sequence $\{a(n)\}$ *nearly* f-subadditive if

$$a(n+m) \leq a(n) + a(m) + f(n+m) \tag{1}$$

holds for all $n \leq m \leq 2n$. They showed that if the error term f is small,

$$\sum_{n=1}^{\infty} f(n)/n^2 \quad \text{is finite,} \tag{2}$$

then the limit $\{a(n)/n\}$ still exists. We show that the deBruijn-Erdős condition (2) for the error term is not only sufficient but also necessary in the following strong sense. If $\sum_{n=1}^{\infty} f(n)/n^2 = \infty$, then there exists a nearly f-subadditive sequence $\{b(n)\}$ such that the sequence of slopes $\{b(n)/n\}$ takes every rational number. On the other hand, we show that their condition can be weakened such that the limit exists if (1) holds only for the pairs $n \leq m \leq cn$ for some fixed $c > 1$. (Received August 20, 2018)