The Leray transform is the most natural higher dimensional analogue of the familiar Cauchy transform in one complex variable. But unlike its one dimensional cousin, the most usual presentation of Leray makes explicit reference to the underlying domain. We will present an alternative, universal description in terms of projective dual coordinates. This leads to the definition of projective dual CR-structures and a corresponding pair of canonical Hardy spaces associated to any strongly C-convex hypersurface. The Leray transform is shown to factor through orthogonal projection onto the conjugate dual Hardy space. These results will be demonstrated with explicit computations on a family of model hypersurfaces. (Received August 20, 2018)