There are two classical methods for constructing solutions of the Korteweg-de Vries (KdV) equation. The inverse scattering method (ISM) is used in the rapidly decaying case, and relates the initial conditions to the spectral data of the associated linear Schroedinger operator. The finite-gap method, on the other hand, produces solutions that are periodic or quasi-periodic.

It has long been understood that the two methods should be related, but a precise description of such a relation was lacking. A key difference between the two methods is that the finite-gap method is symmetric with respect to spatial involution, while the ISM is not.

We present a generalization of the inverse scattering method for the one-dimensional Schroedinger equation that is symmetric with respect to spatial involution. This method specializes to both the classical ISM and to the finite-gap method. The corresponding solutions of the KdV equation are bounded, but not rapidly decaying at infinity. (Received August 21, 2018)