Beyond super-resolution. Preliminary report.

The problem of super-resolution in general terms is to recuperate a finitely supported measure $\mu$ given finitely many of its coefficients $\hat{\mu}(k)$ with respect to some orthonormal system in the case when the number of coefficients required is substantially smaller than a power of the minimal separation among the points in the support of $\mu$. In this paper, we consider the more severe problem of recuperating $\mu$ approximately without any assumption on $\mu$ beyond having a finite total variation. In particular, the minimal separation of the support of $\mu$ is 0. A variant of this problem is of interest in machine learning as well as the inverse problem of de-convolution. We define an appropriate notion of a distance between the target measure and its recuperated version, give an explicit expression for the recuperation operator, and estimate the distance between $\mu$ and its approximation. We show that these estimates are the best possible in many different ways. In particular, we offer one explanation as to why an approximation of a finitely supported measure is bounded from below if the amount of information is smaller than what is demanded in the super-resolution problem. (Received August 20, 2018)