Fix an irrational number $A$, and consider the action of the group of pairs of integers on the real line defined as follows: the pair $(m, n)$ sends a point $x$ to $x + m + nA$. Since the orbits of this action are dense, the quotient topology on the orbit space is trivial and continuous real-valued functions are constant. Can we give the space any type of useful "smooth" group structure?

The answer is "yes": its natural diffeological group structure. It turns out this group is not just some pathological example, but has many interesting associated structures, and is of interest to many areas of mathematics. In particular, it shows up in geometric quantisation and the integration of certain Lie algebroids as the structure group of certain principal bundles, the main topic of this talk.

We will perform Milnor’s construction in the realm of diffeology to obtain a diffeological classifying space for a diffeological group $G$, such as the irrational torus. After mentioning a few hoped-for properties, we then construct a connection 1-form on the $G$-bundle $EG \to BG$, which will naturally pull back to a connection 1-form on sufficiently nice principal $G$-bundles. We then look at what this can tell us about irrational torus bundles. (Received August 20, 2018)