A family of self-similar and translation-invariant random sup-measures with long-range dependence are investigated. They are shown to arise as the limit of the empirical random sup-measure of a stationary heavy-tailed process, inspired by an infinite urn scheme, where same values are repeated at several random locations. The random sup-measure reflects the long-range dependence nature of the original process, and in particular characterizes how locations of extremes appear as long-range clusters represented by random closed sets. A limit theorem for the corresponding point-process convergence is established. (Received August 16, 2018)