Let $A$ be the adjacency matrix of an Erdős–Rényi graph $G$ on $N$ vertices with expected edge density $p$. That is, $A$ is a random $N \times N$ symmetric matrix with zeros on the diagonal and iid Bernoulli($p$) entries above the diagonal. We focus on the sparse regime where $p \sim N^{-c}$ for a fixed constant $c \in (0,1)$ as $N \to \infty$. We determine the asymptotic rate function for deviations of the $k$th moment of $A$ above a fixed multiple of its expectation, for each $k \geq 3$, assuming $c < 1/2$ when $k \geq 4$ and $c < 1/3$ when $k = 3$. The case $k = 3$ gives the sharp upper tail for triangle counts in $G$, extending a previous result of Eldan holding for $c < 1/18$. We also obtain results for large deviations of general subgraph counts in $G$ (for narrower ranges of $c$), as well as for a general class of spectral statistics of $A$ that includes the Perron–Frobenius eigenvalue. (Received August 21, 2018)