Truncated shifted Yangians are algebras which quantize slices to spherical Schubert varieties in the affine Grassmannian of a semisimple group $G$. In recent work with Kamnitzer, Tingley, Webster and Yacobi, we have shown that the representation theory of these algebras can be described using KLR algebras. This connection was made possible by the recently introduced theory of Coulomb branches for 3d $N = 4$ gauge theories, and has applications in categorical and geometric representation theory. In this talk we will overview these topics, and also discuss applications to some slightly more classical problems in representation theory, such as the study of Gelfand-Tsetlin modules and category $O$ for Yangians. (Received January 27, 2019)