The firing patterns of neurons in sensory systems give rise to combinatorial codes, i.e. subsets of the boolean lattice. These firing patterns represent the abstract intersection patterns of subsets of a Euclidean space, and an open problem is identifying the combinatorial properties of neural codes which distinguish the geometric properties of the corresponding subsets. We introduce the polar complex, a simplicial complex associated to any combinatorial code, and use this complex and its associated Stanley-Reisner ideal to to identify some distinguishing characteristics of codes arising from feed-forward neural networks. We demonstrate the polar complex of such codes is shellable, and make connections to other questions in the study of boolean functions. (Received January 25, 2019)