Circular Flows in Planar Graphs.

For integers $a \geq 2b > 0$, a circular $a/b$-flow is a flow that takes values from $\{\pm b, \pm(b + 1), \ldots, \pm(a - b)\}$. The Planar Circular Flow Conjecture states that every $2k$-edge-connected planar graph admits a circular $(2 + \frac{2}{k})$-flow. The cases $k = 1$ and $k = 2$ are equivalent to the Four Color Theorem and Grötzsch’s 3-Color Theorem. For $k \geq 3$, the conjecture remains open. Here we make progress when $k = 4$ and $k = 6$. We prove that (i) every 10-edge-connected planar graph admits a circular $5/2$-flow and (ii) every 16-edge-connected planar graph admits a circular $7/3$-flow. The dual version of statement (i) on circular coloring was previously proved by Dvořák and Postle (Combinatorica 2017), but our proof has the advantages of being much shorter and avoiding the use of computers for case-checking. Further, it has new implications for antisymmetric flows. Statement (ii) is especially interesting because the counterexamples to Jaeger’s original Circular Flow Conjecture are 12-edge-connected nonplanar graphs that admit no circular $7/3$-flow. Thus, the planarity hypothesis of (ii) is essential. (Received January 03, 2019)