Let $R$ be a commutative noetherian ring. Results of Gruson-Raynaud, Jensen, and Osofsky show that if $d$ is a nonnegative integer such that either $R$ has Krull dimension at most $d$ or $|R| \leq \aleph_d$, then every flat $R$-module has projective dimension at most $d$. In particular, if either of these conditions holds, then every localization of $R$ has finite projective dimension.

It is natural to ask whether the same conclusion holds more generally. We give a negative answer to this question by showing how a construction of Nagata produces a commutative noetherian ring such that every localization at a prime ideal has infinite projective dimension. The ring is particularly nice, being an integral domain such that every localization at a prime ideal is regular. (Received January 08, 2019)