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Luis Montejano* (luis@im.unam.mx), Instituto de Matematicas, UNAM, Campus Juriquilla,
Boulevard Juriquilla 3001, 76230 Queretaro, Mexico. *On the real geometric hypothesis of Banach.*

The following is known as the geometric hypothesis of Banach: let V be an m -dimensional Banach space with unit ball B and suppose all n -dimensional subspaces of V are isometric (all the n -sections of B are affinely equivalent). In 1932, Banach conjectured that under this hypothesis V is isometric to a Hilbert space (the boundary of B is an ellipsoid). Gromov proved in 1967 that the conjecture is true for n =even and Dvoretzky derived the same conclusion under the hypothesis m =infinity. We prove this for $n=5$ and 9 and give partial results for an integer $n=4k+1$. The ingredients of the proof are classical homotopic theory, irreducible representations of the orthogonal group and convex geometry. Suppose B is an $(n+1)$ -dim. convex body with the property that all its n -sections through the origin are affinity equivalent to a fixed n -dimensional body K . Using the characteristic map of the tangent vector bundle to the n -sphere, it is possible to prove that if n =even, then K must be a ball and using homotopical properties of the irreducible subgroups of $SO(5)$ and $SO(9)$, we prove that if $n=5,9$, then K must be a body of revolution. Finally, we prove that, if this is the case, then there must be a section of B which is an ellipsoid and consequently B must be also an ellipsoid. (Received January 28, 2019)