Monotone rank of a matrix is the minimal possible rank of a matrix obtained by applying arbitrary monotone functions to each row of $M$. The problem of finding the non-linear rank often arises in neuroscience context. The motivations for the monotone-rank problem is two-fold: 1) "Inverting" a 1-layer neural network, i.e. find the minimal number of the neurons in the input layer if you are given an output of the second layer. and, more generally, 2) estimating the dimension of the space of stimuli, sampled by neurons. While an exact algorithm for computing the nonlinear rank is unknown, it can be efficiently estimated using topological methods, as it is closely related to the geometry and topology of hyperplane arrangements.

A natural tool that captures the essence of the problem is directed complexes – a generalization of simplicial complexes that allows us to keep track of the order of vertices. Directed complexes capture much of the relevant geometric information. For example, the nonlinear rank, as well as other geometric properties of data can be estimated from the homology of an associated directed complex. I will present results and conjectures about the directed complexes associated to some problems of estimating the dimension of the space of stimuli. (Received January 25, 2019)