We derive asymptotic results for the magnitude of contagion in random directed networks and give an analytical expression for the asymptotic fraction of infected nodes, in terms of network characteristics. Our results extend previous studies on contagion in random graphs to inhomogeneous directed graphs with a given degree sequence and arbitrary distribution of weights. We also introduce a criterion for the resilience of a directed network and quantify how contagion amplifies small shocks to the network. We then study the problem of optimal investment in security, under network contagion risk. (Received January 27, 2019)