Asymptotically efficient simulation of elliptic problems with small random forcing.

Recent rare-event simulations show that the large deviation principle (LDP) for stochastic problems plays an important role in both theory and simulation, for studying rare events induced by small noise. Practical challenges of applying this useful technique include minimizing the rate function numerically and incorporating the minimizer into the importance sampling scheme for the construction of efficient probability estimators. For a spatially extended system where the noise is modeled as a random field, even for simple steady state problems, many new issues are encountered in comparison to the finite dimensional models. We consider the Poisson equation subject to a Gaussian random forcing with vanishing amplitude. In contrast to the simplified rate functional given by space white noise, we consider the covariance operator of trace class such that the effects of small noise of moderate or large correlation length on rare events can be studied. We have constructed an LDP-based importance sampling estimator with a sufficient and necessary condition to guarantee the weak efficiency, where numerical approximation of the large deviation principle is also addressed. Numerical results will be presented. (Received January 28, 2019)