For a rational polynomial $f$ and rational numbers $c, u$, we put $f_c(x) := f(x) + c$, and consider the Zsigmondy set $Z(f_c, u)$ associated to the sequence $\{f_c^n(u) - u\}_{n \geq 1}$, where $f_c^n$ is the $n$-st iteration of $f_c$. In this paper, we prove that if $u$ is a rational critical point of $f$, then there exists an $M_f > 0$ such that $M_f \geq \max_{c \in \mathbb{Q}} \{Z(f_c, u)\}$. (Received August 16, 2019)