Wild character varieties parametrize monodromy representations of flat meromorphic connections on compact Riemann surfaces. They are classical objects with remarkable geometric and topological properties.

I will recall how intrinsic geometric structures resolve singularities of wild character varieties and will show that known algebraic symplectic structures extend naturally to the resolutions. This is based on a new universal method for producing symplectic structures which is a joint work with Arinkin and Toën. I will also describe recent joint works with Chuang, Diaconescu, Donagi, and Nawata which extract cohomological invariants of wild character varieties from enumerative Calabi-Yau geometry and refined Chern-Simons invariants of torus knots. (Received August 18, 2019)