Finite $W$-algebras are intimately related to the geometry of nilpotent orbits and the infinite dimensional representation theory of Lie algebras. They are defined in terms of a nilpotent orbit, and they are an invariant subalgebra of a left ideal in universal enveloping algebra of a reductive Lie algebra. Outside of type A there is no known formula for calculating generators of these algebras (apart from specific examples). Recent work by Kac, De Sole, and Valeri has resulted in a Lax type operator which produces generators of an important subalgebra of a finite $W$-algebra, and in this work we extend their results. This results in a formula for generators of important classes of classical finite $W$-algebras. (Received August 19, 2019)