The famous Turán’s inequality [1] asserts that, if $P(z)$ is a polynomial of degree $n \geq 1$ having all its zeros in $|z| \leq 1$ then

$$\max_{|z|=1} |P'(z)| \geq \left( \frac{n}{2} \right) \max_{|z|=1} |P(z)|.$$ 

This result is best possible and equality holds for any polynomial which has all its zeros on $|z| = 1$.

Turán’s inequality restricts the zeros on and within the unit disc. Now naturally a question will arise; what is the analogous interpretation if some of the zeros of $P(z)$ lie outside the unit circle? Here we make an attempt to answer this with some special class of polynomials. These results look quite natural and fundamental in nature.

References


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