Existence, continuation, and lower mass bounds for the Landau equation.

Kinetic equations model gas and particle dynamics, specifically focusing on the interactions between the micro-, meso-, and macroscopic scales. Mathematically, they demonstrate a rich variety of nonlinear phenomena, such as hypoellipticity through velocity averaging and Landau damping. The question of well-posedness remains an active area of research.

In this talk, we look at the Landau equation, a mathematical model for plasma physics arising from the Boltzmann equation as so-called grazing collisions dominate. Previous results are in the perturbative regime, or in the homogeneous setting, or rely on strong a priori control of the solution (the most crucial assumption being a lower bound on the density, as this prevents the elliptic terms from becoming degenerate).

We prove that the Landau equation has local-in-time solutions with no additional a priori assumptions; the initial data is even allowed to contain regions of vacuum. We then prove a ”mass spreading” result via a probabilistic approach. This is the first proof that a density lower bound is generated dynamically from collisions. From the lower bound, it follows that the local solution is smooth, and we establish the mildest (to date) continuation criteria for the solution to exist for all time. (Received August 16, 2019)