A domain exchange map (DEM) is a dynamical system defined on a smooth Jordan domain which is a piecewise translation. We explain how to use cut-and-project sets to construct minimal DEMs. Specializing to the case in which the domain is a square and the cut-and-project set is associated to a Galois lattice, we construct an infinite family of DEMs in which each map is associated to a PV number. We develop a renormalization scheme for these DEMs. Certain DEMs in the family can be composed to create multistage, renormalizable DEMs. (Received August 15, 2019)