

1152-03-489

Jack H Lutz* (lutz@iastate.edu) and **Elvira Mayordomo**. *The hyperspace dimension theorem.*

We prove a general *hyperspace dimension theorem*. Let X be a separable metric space, and let $\mathcal{K}(X)$ be the *hyperspace* of X , i.e., the set of all nonempty compact subsets of X endowed with the Hausdorff metric. For each gauge family (Hausdorff family of gauge functions) φ , we define a *jump* $\tilde{\varphi}$ of φ that is also a gauge family. Our theorem says that, for every subset E of X , the $\tilde{\varphi}$ -gauged packing dimension of $\mathcal{K}(E)$ in $\mathcal{K}(X)$ is at most the φ -gauged packing dimension of E in X . A few very special corollaries of this theorem were previously known.

The logical structure of our proof is of particular interest. We first extend two algorithmic fractal dimensions—computability-theoretic versions of classical Hausdorff and packing dimensions that assign dimensions $\dim(x)$ and $\text{Dim}(x)$ to *individual points* $x \in X$ —to arbitrary separable metric spaces and to arbitrary gauge families. We then extend the point-to-set principle of J. Lutz and N. Lutz (2018) in these same two ways, and we use this principle to prove the theorem. (Received September 10, 2019)