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**Eric Rowland\*** ([eric.rowland@hofstra.edu](mailto:eric.rowland@hofstra.edu)) and **Reem Yassawi**. *Restricted Lucas congruences for Apéry numbers modulo  $p^2$* . Preliminary report.

In 1982 Ira Gessel showed that the Apéry numbers  $A(n) = \sum_{k=0}^n \binom{n}{k}^2 \binom{n+k}{k}^2$  satisfy the Lucas congruence

$$A(n) \equiv A(n_0)A(n_1) \cdots A(n_\ell) \pmod{p},$$

where  $n_\ell \cdots n_1 n_0$  is the standard base- $p$  representation of  $n$ . Additionally, Gessel proved a Lucas congruence modulo 9:

$$A(n) \equiv A(n_0)A(n_1) \cdots A(n_\ell) \pmod{9},$$

where  $n_\ell \cdots n_1 n_0$  is the base-3 representation of  $n$ . For larger primes  $p$ , the analogous congruence modulo  $p^2$  does not hold in general. However, if we restrict to certain sets of base- $p$  digits, then we do obtain valid congruences modulo  $p^2$ . For example, if each digit in the base-5 representation of  $n$  belongs to  $\{0, 2, 4\}$ , then

$$A(n) \equiv A(n_0)A(n_1) \cdots A(n_\ell) \pmod{25}.$$

For each prime  $p$ , we identify a set of base- $p$  digits that supports a restricted Lucas congruence for the Apéry numbers modulo  $p^2$ . These congruences were discovered experimentally, and their existence is partially explained by yet another result of Gessel. (Received September 03, 2019)