Mazur proved that, given an elliptic curve $E/\mathbb{Q}$, there are only 15 possible torsion structures for the Mordell-Weil group $E(\mathbb{Q})$. Many generalizations of this theorem have been obtained upon replacing $\mathbb{Q}$ with a number field $F/\mathbb{Q}$. More recently, there has been much interest in studying the same question when $E/\mathbb{Q}$ is base-changed to an infinite extension $F/\mathbb{Q}$, such as the compositum of all quadratic or cubic extensions of $\mathbb{Q}$. In this talk we study what happens when changing base to the compositum of all number fields with Galois group $G$ for a fixed group $G$. We start with a survey of what is known and then continue studying the problem by giving a group theoretic condition, called generalized $G$-type, which is a necessary condition for a number field with Galois group $H$ to be contained in that compositum. In general, group theory allows one to reduce the original problem to the question of finding rational points on finitely many modular curves. To illustrate this method we completely determine which torsion structures occur for elliptic curves defined over $\mathbb{Q}$ and base-changed to the compositum of all fields whose Galois group is $A_4$. (Received January 28, 2019)