In this talk I will discuss new results about harmonic functions which vanish continuously on an open, convex subset of the boundary. These results stem from new Minkowski-type estimates on the “generalized critical set,” which includes the critical set in the interior, the geometric singular set of the boundary, and the set where the normal derivative vanishes on the boundary. Such estimates give new insight into the size and structure of the critical set as it approaches the boundary, as well as strongly improves upon previous results on the size and structure of the set where the normal derivative vanishes on the boundary. (Received January 27, 2019)