We prove an analogue of a perturbation result for the Dirichlet problem of divergence form elliptic operators by Fefferman, Kenig and Pipher, for the degenerate elliptic operators of David, Feneuil and Mayboroda, which were developed to study geometric and analytic properties of sets with boundaries whose co-dimension is higher than 1. These operators are of the form $\mathbf{-divA}\mathbf{\nabla}$, where $A$ is a degenerate elliptic matrix carefully crafted to weigh the distance to the high co-dimension boundary in a way that allows the nourishment of an elliptic theory. When this boundary is a $d$–Alhfgors-David regular set, we prove that the membership of the harmonic measure in $A_\infty$ is preserved under suitably small Carleson-measure perturbations of the degenerate elliptic matrix $A$, yielding in turn that the $L^p$–solvability of the Dirichlet problem is also stable under these perturbations (with possibly different $p$). The method of proof follows an analogue of the extrapolation technique of Hofmann and Martell applied to divergence form elliptic operators, and the use of projection operators to sawtooth domains. This is joint work with Svitlana Mayboroda. (Received February 03, 2019)