We study two classical problems in convex geometry in $\mathbb{R}^n$ associated to $\mathcal{A}$-harmonic PDEs, quasi-linear elliptic PDEs whose structure is modeled on the $p$-Laplace equation. Let $p$ be fixed with $2 \leq n \leq p < \infty$. For a convex compact set $E \subset \mathbb{R}^n$ we define and then prove the existence and uniqueness of a $\mathcal{A}$-harmonic Green's function $G$ for the complement of $E$ with pole at infinity. Then we define a quantity $C_{\mathcal{A}}(E)$ which can be seen as the behavior of $G$ near infinity, using this quantity in the place of capacity we prove a Brunn-Minkowski inequality for $C_{\mathcal{A}}$.

We also consider the Minkowski problem for a measure associated with $G$, we show that the necessary and sufficient conditions for existence are the same as in the classical Minkowski problem. (Received January 26, 2019)