Stability of graphical tori with almost nonnegative scalar curvature.

By works of Schoen–Yau and Gromov–Lawson any Riemannian manifold with nonnegative scalar curvature and diffeomorphic to a torus is isometric to a flat torus. Gromov conjectured subconvergence of tori with respect to a weak Sobolev type metric when the scalar curvature goes to 0. We prove flat and intrinsic flat subconvergence to a flat torus for sequences of 3-dimensional tori $M_j$ that can be realized as graphs of functions defined over flat tori satisfying a uniform upper diameter bound, a uniform lower bound on the area of the smallest closed minimal surface, and scalar curvature bounds of the form $R_{g_{M_j}} \geq -1/j$. We also show that the volume of the manifolds of the convergent subsequence converges to the volume of the limit space. We do so adapting results of Huang–Lee, Huang–Lee–Sormani and Sormani. Furthermore, our results also hold when the condition on the scalar curvature of a torus $(M, g_M)$ is replaced by a bound on the quantity $-\int_T \min\{R_{g_M}, 0\} d_{g_T}$, where $M = graph(f)$, $f : T \to \mathbb{R}$ and $(T, g_T)$ is a flat torus. (Received January 30, 2019)