In this lecture, I will describe the recent solution of an old problem originating in mathematical physics, namely that of proving the existence of the large dimension limits of the Harish-Chandra/Itzykson-Zuber and Brezin-Gross-Witten matrix integrals, and realizing these limits as generating functions for a class of planar combinatorial structures. The problem itself is captivating: while of an ostensibly analytic nature, ideas from combinatorics, geometry, representation theory, and probability are needed to break through to a solution. So, I hope that this lecture will be appealing to those who enjoy seeing interactions between different parts of mathematics. But more than this, I will candidly discuss the ups and downs, the successes and defeats, encountered along the path to a solution - experiences which, I believe, are universal among mathematicians, and which ultimately make our discipline so rewarding. (Received August 15, 2019)