The famous Szemerédi-Trotter theorem states that any arrangement of $n$ points and $n$ lines in the plane determines $O(n^{4/3})$ incidences, and this bound is tight. In this talk, we present some Turán-type results for point-line incidences.

Let $L_1$ and $L_2$ be two sets of $t$ lines in the plane and let $P = \{\ell_1 \cap \ell_2 : \ell_1 \in L_1, \ell_2 \in L_2\}$ be the set of intersection points between $L_1$ and $L_2$. We say that $(P, L_1 \cup L_2)$ forms a natural $t \times t$ grid if $|P| = t^2$, and $\text{conv}(P)$ does not contain the intersection point of some two lines in $L_i$, for $i = 1, 2$. For fixed $t > 1$, we show that any arrangement of $n$ points and $n$ lines in the plane that does not contain a natural $t \times t$ grid determines $O(n^{4/3-\varepsilon})$ incidences, where $\varepsilon = \varepsilon(t)$. We also provide a construction of $n$ points and $n$ lines in the plane that does not contain a natural $2 \times 2$ grid and determines at least $\Omega(n^{1+\frac{1}{14}})$ incidences. Joint work with Andrew Suk. (Received August 08, 2019)