We study the geometry and topology of a certain family of exotic Springer fibers from an explicit, diagrammatic point of view. These algebraic varieties appear as the fibers under a resolution of singularities of the exotic nilpotent cone which plays a prominent role in Kato’s Deligne–Langlands type classification of simple modules for multiparameter Hecke algebras of type C. We describe our results in terms of the combinatorics of the two-boundary Temperley–Lieb algebra. This provides the general framework to construct geometric versions of Khovanov’s arc algebra arising from exotic Springer fibers as well as homological knot invariants in thickened surfaces with two punctures. (Received August 31, 2019)