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We develop a quantitative analogue of equational reasoning which we call quantitative algebra. We define an equality relation indexed by rationals:  $a =_\varepsilon b$  which we think of as saying that “ $a$  is approximately equal to  $b$  up to an error of  $\varepsilon$ ”. The well-known connection between equational theories and monads on the category  $\text{Set}$  extends to a correspondence between quantitative equational theories and monads on the category of extended metric spaces and non-expansive maps. Free algebras in this case come with a metric and the algebra operations are non-expansive.

We have 4 interesting examples where we have a quantitative equational theory whose free algebras correspond to well known structures. In each case we have finitary and continuous versions. The four cases are: Hausdorff metrics from quantitative semilattices; Kantorovich metrics from barycentric algebras and also from pointed barycentric algebras and, finally, the total variation metric on the probability distributions on a metric space from a variant of barycentric algebras. It is interesting that the Kantorovich (often called the Wasserstein) metric emerges in this natural way from equations that say nothing about probability distributions. (Received August 01, 2019)