Iwasaki proposed the study of the structure of a finite group based on its number of ordinary real-valued irreducible characters. It is well-known that a finite group has a unique such character if and only if the group has odd order, and Iwasaki proved that if a finite group has exactly two such characters, then it must have a normal Sylow 2-subgroup with specific structure. Further, Moretó and Navarro studied the structure of a finite group with exactly 3 real-valued irreducible characters, and Tong-Viet studied the structure when there are 4 real-valued irreducible characters.

If $G$ is a finite group, let $\text{Sol}(G)$ denote the largest solvable normal subgroup of $G$. Here, we first prove that if $G$ has at most 5 real-valued irreducible characters, then $G/\text{Sol}(G)$ must be isomorphic to one group in a specific list. Further, we prove that there exists a function $f$ from the positive integers to the positive integers such that, if $G$ is a finite group with at most $k$ real-valued irreducible characters, then $|G/\text{Sol}(G)| \leq f(k)$. (Received August 27, 2019)