
The well-known results of Barlow and Bass that establish the existence and regularity of random walks on generalized Sierpinski carpets imply that functions which are smooth (in the sense that applying any power of the Laplacian gives a continuous function) are Hölder continuous with exponent $d_W - d_H$, where these exponents are the “walk dimension” $d_W$ of the random process and the Hausdorff dimension $d_H$ of the underlying set.

Recent work of Alonso-Ruiz, Baudoin, Chen, Rogers, Shanmugalingam and Teplyaev on functions of bounded variation on metric measure spaces gives reason to believe that this Hölder exponent can be improved, and that the optimal Hölder exponent is $d_W - d_H + d_{sep}$, where $d_{sep}$ is the “separating dimension”: the minimal Hausdorff dimension such that there is a basis for the topology in which all neighborhoods have boundary with this dimension. (One has $d_{sep} = d_{tH} - 1$, where $d_{tH}$ is the “topological Hausdorff dimension”.)

We discuss several reasons for believing $d_W - d_H + d_{sep}$ should be the optimal Hölder estimate in this context and present some numerical evidence for its validity that was developed in joint work with Canner, Hayes, Huang, Orwin and Teplyaev. (Received September 02, 2019)