We consider the Cauchy problem for the Schrodinger maps evolution when the domain is the hyperbolic plane. An interesting feature of this problem compared to the more widely studied case on the Euclidean plane is the existence of a rich new family of finite energy harmonic maps. These are stationary solutions, and thus play an important role in the dynamics of Schrodinger maps. The main result of this article is the asymptotic stability of (some of) such harmonic maps under the Schrodinger maps evolution. More precisely, we prove the nonlinear asymptotic stability of a finite energy equivariant harmonic map $Q$ under the Schrodinger maps evolution with respect to non-equivariant perturbations, provided that $Q$ obeys a suitable linearized stability condition. (Received August 30, 2019)