Operator integration is a powerful tool enabling analysis of functions with noncommutative arguments. Such functions arise, for example, in matrix analysis, mathematical physics, noncommutative geometry, and statistical estimation. Over some seventy years of its development, theory underlying multilinear operator integration has accumulated many deep results and important applications. We will discuss major advancements made in recent years and their impact on differentiation and approximation of operator functions. (Received August 14, 2019)