Building on previous work of Rozansky and Willis, we generalise Rasmussen’s $s$-invariant to $\#^r(S^1 \times S^2)$. Such an $s$-invariant can be computed by approximating the Khovanov-Lee complex of a link in $\#^r(S^1 \times S^2)$ with that of appropriate links in $S^3$. We use the approximation result to compute the $s$-invariant of a family of links in $S^3$ which seems otherwise inaccessible, and use this computation to deduce an adjunction inequality for null-homologous surfaces in a (punctured) connected sum of $\mathbb{CP}^2$. This inequality has several consequences: first, the $s$-invariant of a knot in $S^3$ does not increase under the operation of adding a null-homologous full twist. Second, the $s$-invariant cannot be used to distinguish $S^4$ from homotopy 4-spheres obtained by Gluck twist on $S^4$. We also prove a connected sum formula for the $s$-invariant of links in $S^3$, improving a previous result of Beliakova and Wehrli. We define two $s$-invariants for links in $S^1 \times S^2$. One of them gives a lower bound to the slice genus in $S^1 \times B^3$ and the other one to the slice genus in $D^2 \times S^2$. Lastly, we give a combinatorial proof of the slice Bennequin inequality in $S^1 \times S^2$. (Received August 25, 2019)