Torus bundles that bound rational homology circles.

Classifying rational homology 3-spheres that bound rational homology 4-balls is a widely explored question. One way to construct such rational homology 3-spheres is by attaching a 2-handle to a rational homology $S^1 \times D^3$ in a way that results in a rational homology 4-ball. Recent work of Akbulut-Larson, for example, used this method to find new examples of Brieskorn spheres that bound rational homology 4-balls. A simple family of rational $S^1 \times S^2$s consists of the torus bundles over $S^1$ with first Betti number equal to 1. In this talk, we will construct torus bundles that bound rational homology $S^1 \times D^3$s and use them to construct rational homology 3-spheres that bound rational homology 4-balls. We will then use lattice theory and Heegaard Floer homology correction terms to obstruct certain torus bundles from bounding rational homology $S^1 \times D^3$s and discuss the limitations of these obstructions in giving a complete classification of torus bundles that bound rational $S^1 \times D^3$s. This is a work in progress. (Received August 31, 2019)