1150-03-207 **Iskander Sh. Kalimullin** and **Russell Miller*** (russell.miller@qc.cuny.edu), Mathematics Dept., Queens College - CUNY, 65-30 Kissena Blvd., Queens, NY 11367, and **Hans Schoutens**.
Degrees of transcendence bases of computable fields.

Downey, Lempp, and Wu showed that the only Turing degrees $\boldsymbol{c}$ for which the adjacency relation on a linear order can be intrinsically of degree $\boldsymbol{c}$ are the degrees $\boldsymbol{c} = \mathbf{0}$ (for finite linear orders) and $\boldsymbol{c} = \mathbf{0'}$ (by an earlier result of Downey and Moses). We consider the same question for the natural Π_1 relations of transcendence and algebraic independence on computable fields.

Fields prove to be more supple than linear orders. For every computably enumerable degree $\boldsymbol{c}$, these relations can both be intrinsically of degree $\boldsymbol{c}$. It is also possible for the spectra of these relations to be the upper cone of c.e. degrees above an arbitrary Δ_2 degree $\boldsymbol{c}$, or to equal the set of c.e. degrees that can enumerate an arbitrary Σ_2 set S. Since the degree of the independence relation on a field is always the least degree of a transcendence basis for that field, these results describe the possible degrees of transcendence bases for these fields. In particular, this is the first proof that a computable field exists for which no computable copy has a computable transcendence basis. The proofs use algebraic curves of positive genus to code information into the transcendence and independence relations. (Received July 09, 2019)