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**Peter A Cholak\*** (cholak@nd.edu), 255 Hurley, Department of Math, Notre Dame, IN 46556.

*Thin sets and the Preservation of Hyperimmunities.* Preliminary report.

Let  $c$  be a coloring of  $n$ -tuples (of  $\mathbb{N}$ ) by finitely many colors. For  $l$  less than the number of colors, a set  $T$  is  $l$ -thin iff  $c$  uses at most  $l$  colors to color all the  $n$ -tuples from  $T$ . The statement such a thin set exists for all colorings is called  $RT_{<\infty,l}^n$ . A function  $f : \mathbb{N} \rightarrow \mathbb{N}$  is *hyperimmune* if it is not dominated by any computable function. A problem  $\mathbf{P}$  admits the preservation of  $p$  hyperimmunities if, for every set  $Z$  and every collection  $\{f_s : s < p\}$  of  $Z$ -hyperimmune functions, every instance  $\mathcal{I}_{\mathbf{P}} \leq Z$  of  $\mathbf{P}$  has a solution  $\mathcal{S}_{\mathbf{P}}$  such that, for every  $s < p$ ,  $f_s$  is  $Z \oplus \mathcal{S}_{\mathbf{P}}$ -hyperimmune. If the property holds for every  $\mathbf{P}$ -instance, that is, not only  $Z$ -computable instances, then we say that  $\mathbf{P}$  admits *strong* preservation of  $p$  hyperimmunities.

**Conjecture:**  $RT_{<\infty,p^n C_n}^n$  admits the strong preservation  $p$  hyperimmunities and, moreover, this bound is tight, where  $C_n$  is the  $n$ th Catalan number.

In this talk we will discuss this conjecture and other related results and conjectures. This is joint work with Ludovic Patey. (Received July 11, 2019)