Given a function $T : X \to X$ on a topological space $X$ it is natural to study the set of $x \in X$ such that the forward orbit

$$\{T^n(x) : n \in \mathbb{N}\}$$

of $x$ under $T$ has some specified property. For instance, one might study the set of $x \in X$ such that the forward orbit of $x$ is dense in $X$.

In my talk, I will consider two particular cases of this general setup in the context of definable complexity. First, I will discuss a result of mine from 2015 where I show that the set of $x \in [0,1]$ which are normal of order precisely $n$ (for any fixed $n \in \mathbb{N}$) is Wadge-complete for the class of differences of $\Pi^3_0$ sets. Second, I will discuss a recent result (joint with Paul B. Larson), where we show that the set of hypercyclic vectors for a unilateral weighted shift can be made to have arbitrarily high definable complexity.

I will discuss relevant definitions and give background to motivate these results. (Received July 12, 2019)