In 1987 Alavi, Malde, Schwenk and Erdős showed that the independent set sequence of a graph is unconstrained in terms of its pattern of rises and falls, in the following sense: for any \( m \in \mathbb{N} \) and any permutation \( \pi \) of \( \{1, \ldots, m\} \) there is a graph with largest independent set having size \( m \), and with

\[
i_{\pi(1)} \leq i_{\pi(2)} \leq \cdots \leq i_{\pi(m)},
\]

where \( i_k \) is the number of independent sets of size \( k \) in the graph. Their construction yielded a graph with around \( m^{2m} \) vertices, and they raised the following question:

Determine the smallest order large enough to realize every permutation of order \( m \) as the sorted indices of the vertex independent set sequence of some graph.

We answer this question exactly. Alavi et al. also observed that the matching sequence of a graph is, by contrast, quite constrained — at most \( 2^{m-1} \) permutations of \( \{1, \ldots, m\} \) can be realized as the sorted indices of the matching sequence of some graph. They asked whether the upper bound of \( 2^{m-1} \) was optimal; we show that it is not. Many open problems remain in this area.

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